Examples of Partial Derivatives

① $\cos(xe^y)\sin(x-2y) = f(x,y)$

Find f_x, f_y

② $\dfrac{\arctan(y\sqrt{x})}{10} = g(x,y)$

Find g_x, g_y.

③ $h(x,y,z) = \dfrac{y\ln(xz)}{x+z}$

Find h_y, h_z

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① $\cos(xe^y)\sin(x-2y) = f(x,y)$

$$f_x = -\sin(xe^y)\cdot e^y \sin(x-2y) + \cos(xe^y)\cdot\cos(x-2y)$$

$$f_y = -\sin(xe^y)\cdot xe^y \sin(x-2y) + \cos(xe^y)\cdot\cos(x-2y)(-2).$$

② $\dfrac{\arctan(y\sqrt{x})}{10} = g(x,y)$

$(y\sqrt{x})_x$
$= (y x^{1/2})_x$
$= y \frac{1}{2} x^{-1/2}$
$= y \frac{1}{2\sqrt{x}}$.

$$g_x = \frac{1}{10} \frac{1}{1+\underbrace{(y\sqrt{x})^2}_{y^2x}} \left(\frac{y}{2\sqrt{x}} \right)$$

$$g_y = \frac{1}{10} \frac{1}{1+y^2x} \cdot \sqrt{x}$$

③

$$h(x,y,z) = \frac{y\ln(xz)}{x+z}$$

$$h_y = \frac{\ln(xz)}{x+z}$$

$$h_z = \frac{(x+z)[y\ln(xz)]_z - y\ln(xz) \cdot 1}{(x+z)^2}$$

$$= \frac{(x+z)\, y \frac{1}{xz} \cdot x - y\ln(xz)}{(x+z)^2}$$

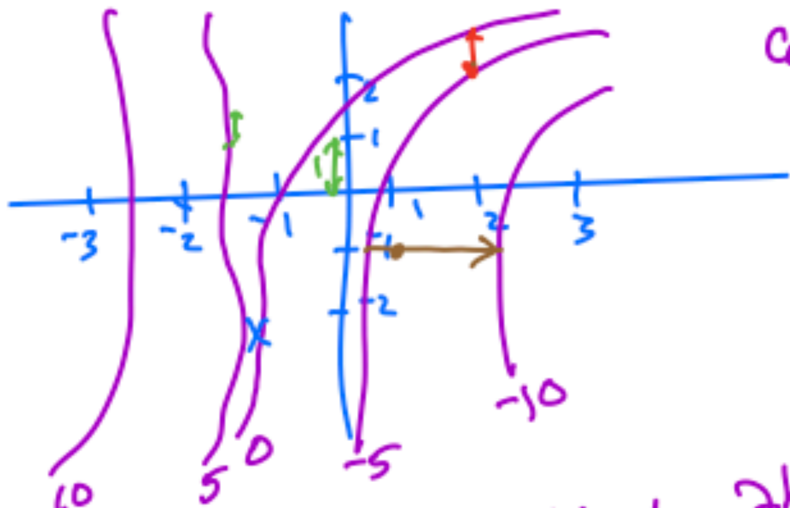
$$= \frac{\frac{y}{z}(x+z) - y\ln(xz)}{(x+z)^2}.$$

$$\left[\arcsin(x)\right]' = \frac{1}{\sqrt{1-x^2}}$$

$$\left[\arccos(x)\right]' = \frac{-1}{\sqrt{1-x^2}}$$

$\frac{\pi}{2} - \arcsin(x)$

Contours and partial deriv.



contour diagram of $h(x,y)$.

Questions: ① Estimate $\frac{\partial h}{\partial x}(1,-1)$

② Estimate $\frac{\partial h}{\partial y}(2,2)$ $\approx \frac{\Delta h}{\Delta y}$

$\approx \frac{0-(-5)}{.6} \approx \frac{5}{.6}$

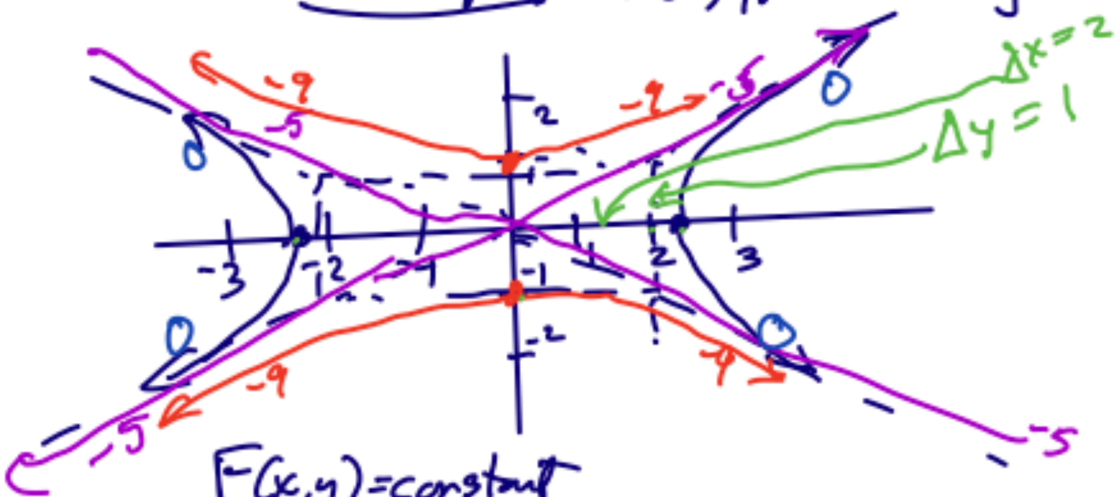
≈ 8.3

$\approx \frac{\Delta h}{\Delta x} \approx \frac{-10-(-5)}{1.5} = \frac{-5}{1.5} = -3.3$

③ Where is $\left|\frac{\partial h}{\partial x}\right|$ maximum? $(-1.3, -2.2)$

Drawing Contour Diagrams

Example: $F(x,y) = x^2 - 4y^2 - 5$



$F(x,y) = \text{constant}$

$F(x,y) = 0 \Rightarrow x^2 - 4y^2 - 5 = 0$

$x^2 - 4y^2 = 5$

$y = 0$

$x = \pm\sqrt{5}$

asymptotes

$x^2 - 4y^2 = 0$

$(x-2y)(x+2y) = 0$

$y = \pm\frac{1}{2}x$

$F(x,y) = -5$

$x^2 - 4y^2 - 5 = -5$

$x^2 - 4y^2 = 0$

$(x-2y)(x+2y) = 0$

$x - 2y = 0 \qquad x + 2y = 0$

$x = 2y \qquad 2y = -x$

$\frac{1}{2}x = y \qquad y = -\frac{1}{2}x$

$F(x,y) = \quad F(0,-1) = 0^2 - 4(-1)^2 - 5$

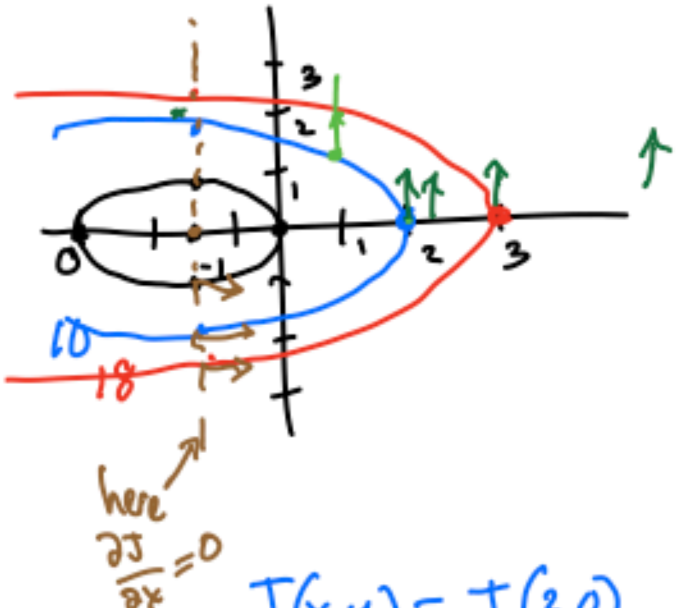
$= -9$

$x^2 - 4y^2 - 5 = -9$

$x^2 - 4y^2 = -4 \Rightarrow \quad 4y^2 - x^2 = 4$

Example: $J(x,y) = x(x+3) + 4y^2$

$$= x^2 + 3x + 4y^2$$



$J(x,y) = 0$

$x^2 + 3x + 4y^2 = 0$

$\underbrace{\left(x + \frac{3}{2}\right)^2}_{x^2+3x+9/4} + \underbrace{4y^2}_{(2y)^2} = \frac{9}{4}$

ellipse centered at $\left(\frac{-3}{2}, 0\right)$

$y = 0 \rightarrow x = 0$ or -3

$x = \frac{-3}{2} \rightarrow y^2 = \frac{9}{16}$ $y = \pm\frac{3}{4}$

$J(x,y) = J(2,0) = 2^2 + 3\cdot 2 = 10$

$x^2 + 3x + 4y^2 = 10$

$\left(x + \frac{3}{2}\right)^2 + 4y^2 = 10 + \overset{2.25}{\frac{9}{4}} = \underset{\frac{49}{4}}{12.25}$

$J(x,y) = J(3,0) = 3^2 + 3\cdot 3 = 18$

$x^2 + 3x + 4y^2 = 18$

$\left(x + \frac{3}{2}\right)^2 + 4y^2 = 18 + \frac{9}{4} = \frac{81}{4} = \left(\frac{9}{2}\right)^2$

At what points (x,y) is it true that $J_y(x,y) = 0$?

① Algebra $J_y = 8y = 0 \Rightarrow y = 0.$

② Contour Diagram: any pt on x-axis

Note $J_x(x,y) = 0$ when $x = \frac{-3}{2}$. $\left(J_x = 2x + 3 = 0 \Rightarrow x = \frac{-3}{2}\right.$.

Tangent lines to contour are horizontal